THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 1

1. Recall that for every set $X$, the cocountable topology is defined by

$$\mathfrak{T} = \{\emptyset, X\} \cup \{G \subset X \mid X \backslash G \text{ is countable}\}$$

   (a) If $X$ is an uncountable set, is the cocountable topology separable?

   (b) * Show that the the cocountable topology is not $C_I$.

2. Let $X = \mathbb{R}$ and $K = \{\frac{1}{n} \mid n \in \mathbb{N}\}$. The $K$-topology $T_K$ is generated by the base

$$B = \{(a, b) \mid a, b \in \mathbb{R}, a < b\} \cup \{(a, b)\backslash K \mid a, b \in \mathbb{R}, a < b\}$$

   (a) Show that $B$ is a base.

   (b) Let $T_l$ be the lower limit topology on $X$. Show that $T_l \not\subset T_K$ and $T_K \not\subset T_l$.

3. Let $(X, \mathfrak{T})$ be a topological space and $A \subset X$. Define the frontier (or the boundary) of $A$ by

   (i) $\operatorname{Frt}(A) = \overline{A} \cap \overline{X \backslash A}$; or

   (ii) $\operatorname{Frt}(A) = \{x \in X \mid \text{for any } U \in \mathfrak{T} \text{ with } x \in U, \text{ we have } U \cap A \neq \emptyset \text{ and } U \cap (X \backslash A) \neq \emptyset.\}$

   (a) Show that $x \in \overline{A}$ if and only if for any $U \in \mathfrak{T}$ with $x \in U$, we have $U \cap A \neq \emptyset$.

   (b) Show that two definitions of frontier are equivalent.

   (c) Show that $A$ is open if and only if $A = \overline{A} \backslash \operatorname{Frt}(A)$.

   (d) Show that $\operatorname{Int}(A) \cap \operatorname{Frt}(A) = \emptyset$.

   (e) Show that $\operatorname{Frt}(A) = \emptyset$ if and only if $A$ is both open and closed.

   (f) * Give an example of a set $A$ with $\operatorname{Frt}(A) \neq \operatorname{Frt}(\operatorname{Frt}(A))$.